DTU Library

The Magnetic Physical Optics Scattered Field in Terms of a Line Integral

Meincke, Peter; Breinbjerg, Olav; Jørgensen, Erik

Published in:
Proceedings of IEEE Antennas and Propagation Society International Symposium

Link to article, DOI:
10.1109/APS.2000.873847

Publication date:
2000

Document Version
Publisher's PDF, also known as Version of record

The Magnetic Physical Optics Scattered Field in Terms of a Line Integral

Peter Meincke, Olav Breinbjerg, and Erik Jørgensen*
Department of Electromagnetic Systems, Building 348
Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark
Email: pme@emi.dtu.dk

Abstract — An exact line integral representation is derived for the magnetic physical optics field scattered by a perfectly electrically conducting planar plate illuminated by a magnetic Hertzian dipole. A numerical example is presented to illustrate the exactness of the line integral representation.

1. Introduction

The transformation of surface integrals representing scattered or diffracted fields into line integrals has achieved much attention in the past decades. In particular, the traditional surface integral of the electric physical optics (PO) scattered field from a perfectly electrically conducting (PEC) planar plate illuminated by an electric Hertzian dipole has been cast into a line integral along the edges of the plate in [1]. The procedure of [1] has been extended in [2] to derive a line integral representation of the electric PO scattered field from a penetrable planar plate illuminated by a plane wave. The main advantage of the line integral representation is that its numerical evaluation in general requires much less computational effort than that of the corresponding surface integral. This is especially true when the source is located close to the plate since the PO current is then highly peaked and the surface integration requires many sampling points to yield a good accuracy. In [1] only the electric PO scattered field was considered. However, in some important cases, e.g., the application of the magnetic field integral equation, it is also necessary to know the magnetic PO scattered field from PEC planar plates. To the knowledge of the authors, a line integral representation of this magnetic field has not yet been published in the literature. In the present paper we derive such a line integral representation and we consider illumination by a magnetic Hertzian dipole.

2. The Line Integral Representation

Consider a planar PEC plate A located in the xy plane of a rectangular xyz coordinate system. The magnetic Hertzian dipole with position vector $\mathbf{r}_S$ is located in the region $z > 0$. The magnetic PO scattered field $\mathbf{H}^{PO}$ at the observation point F with position vector $\mathbf{r}_F$ can be written as [3]

$$\mathbf{H}^{PO}(\mathbf{r}_F) = \mathbf{H}^A(\mathbf{r}_F) - (\bar{\mathbf{I}} - 2\hat{\mathbf{z}}\hat{\mathbf{z}}) \cdot \mathbf{H}^A(\mathbf{r}_I) \qquad (1)$$

where $\bar{\mathbf{I}}$ is the unit dyad, $\mathbf{r}_I$ is the position vector of the image point with respect to the plane of the plate, and (the time factor $\exp(j\omega t)$ is suppressed)

$$\mathbf{H}^A(\mathbf{r}) = \nabla \times \int\limits_A \hat{\mathbf{z}} \times \mathbf{H}^i(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\,dA' + \frac{j}{kZ}\nabla \times \nabla \times \int\limits_A \hat{\mathbf{z}} \times \mathbf{E}^i(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\,dA'. \tag{2}$$

Herein, $G(\mathbf{r},\mathbf{r}') = \exp(-jkR)/(4\pi R)$ with $R = |\mathbf{R}|$ and $\mathbf{R} = \mathbf{r}' - \mathbf{r}$, k is the wave number, and Z the intrinsic impedance. Moreover, $\mathbf{H}^i$ and $\mathbf{E}^i$ are the incident magnetic and electric fields, respectively. Interchanging the curl and integration operators in (2), using the facts that $\nabla G(\mathbf{r},\mathbf{r}') = -\nabla' G(\mathbf{r},\mathbf{r}') = G(\mathbf{r},\mathbf{r}')(jk + 1/R)\hat{\mathbf{R}}$, with $\hat{\mathbf{R}} = R^{-1}\mathbf{R}$, and that G satisfies the homogeneous wave equation when the observation point is not on the plate, and invoking Stoke's extended theorem, (2) can be cast into the Kottler representation of the magnetic field,

$$\mathbf{H}^A(\mathbf{r}) = -\int\limits_A \hat{\mathbf{z}} \cdot \left[G(\mathbf{r},\mathbf{r}')\nabla'\mathbf{H}^i(\mathbf{r}') - \nabla' G(\mathbf{r},\mathbf{r}')\mathbf{H}^i(\mathbf{r}')\right] dA' + \int\limits_\Gamma \hat{\mathbf{t}} \times \mathbf{H}^i(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\,d\Gamma' + \frac{1}{jkZ}\int\limits_\Gamma \hat{\mathbf{t}} \cdot \mathbf{E}^i(\mathbf{r}')\nabla' G(\mathbf{r},\mathbf{r}')\,d\Gamma'. \tag{3}$$

In this expression, Γ denotes the edge of the plate, $\hat{\mathbf{t}}$ is the edge unit tangent vector related to $\hat{\mathbf{z}}$ via the right-hand rule. The surface integral in (3) is transformed into a line integral using [1, (10), (18), (22), (23)] with $\mathbf{E}^i$ replaced by $\mathbf{H}^i$. The result for $\mathbf{H}^A$ is then

$$\mathbf{H}^A(\mathbf{r}) = -\mathbf{H}^i(\mathbf{r})\chi(\mathbf{r}) + \int\limits_\Gamma \hat{\mathbf{t}} \times \mathbf{H}^i(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\,d\Gamma' + \int\limits_\Gamma \hat{\mathbf{t}} \cdot \left[\mathbf{V}(\mathbf{r},\mathbf{r}')\mathbf{H}^i(\mathbf{r}) + \bar{\mathbf{W}}_H(\mathbf{r},\mathbf{r}') + \frac{1}{jkZ}\mathbf{E}^i(\mathbf{r}')\nabla' G(\mathbf{r},\mathbf{r}')\right] d\Gamma'. \tag{4}$$

The transformation from (3) to (4) is valid only if the Hertzian dipole is not located on the surface of the cone with vertex at $\mathbf{r}$ and generators extending from this point to the edge Γ. In (4) $\chi(\mathbf{r}) = 1$ if the Hertzian dipole is located within the above-mentioned cone and zero otherwise. Furthermore, $\mathbf{V}(\mathbf{r},\mathbf{r}') = \hat{\mathbf{p}} \times \hat{\mathbf{R}}/[4\pi R(1 - \hat{\mathbf{R}} \cdot \hat{\mathbf{p}})]$, with $\hat{\mathbf{p}}$ being an arbitrary unit vector in the xy-plane, and

$$\bar{\mathbf{W}}_H(\mathbf{r},\mathbf{r}') = \frac{1}{4\pi}\hat{\mathbf{R}} \times \int\limits_0^1 \nabla\mathbf{H}^i(\tau\mathbf{R})\exp(-jk\tau R)\,d\tau. \tag{5}$$

The expression for the magnetic PO scattered field is finally obtained by inserting

(4) into (1). This yields

$$\mathbf{H}^{PO}(\mathbf{r}_F) = -\mathbf{H}^i(\mathbf{r}_F)\chi(\mathbf{r}_F) + \int_\Gamma \Big\{ \hat{\mathbf{t}} \cdot \Big[\bar{\mathbf{W}}_H(\mathbf{r}_F, \mathbf{r}') - \bar{\mathbf{W}}_H(\mathbf{r}_I, \mathbf{r}') \cdot (\bar{\mathbf{I}} - 2\hat{\mathbf{z}}\hat{\mathbf{z}})$$

$$+ \mathbf{V}(\mathbf{r}_F, \mathbf{r}') \Big(\mathbf{H}^i(\mathbf{r}_F) + \mathbf{H}^i(\mathbf{r}_I) \cdot (\bar{\mathbf{I}} - 2\hat{\mathbf{z}}\hat{\mathbf{z}}) \Big) \Big] + 2\hat{\mathbf{z}}\hat{\mathbf{z}} \cdot \big(\hat{\mathbf{t}} \times \mathbf{H}^i(\mathbf{r}') \big)\, G(\mathbf{r}_F, \mathbf{r}') \Big\} d\Gamma'. \tag{6}$$

This is the sought-after line integral representation of the magnetic PO scattered field. However, an analytical evaluation of the dyadic $\bar{\mathbf{W}}_H$ in (5) is still required. The magnetic Hertzian dipole has the magnetic current density $\mathbf{J}_m(\mathbf{r}) = \boldsymbol{\alpha}_m \delta(\mathbf{r} - \mathbf{r}_S)$ and it radiates the magnetic field

$$\mathbf{H}^i_m(\mathbf{r}') = \frac{-jG(\mathbf{r}', \mathbf{r}_S)}{kZ} \left[\left(-k^2 + \frac{3jk}{\rho} + \frac{3}{\rho^2} \right) \boldsymbol{\alpha}_m \cdot \hat{\boldsymbol{\rho}}\hat{\boldsymbol{\rho}} + \left(k^2 - \frac{jk}{\rho} - \frac{1}{\rho^2} \right) \boldsymbol{\alpha}_m \right], \; \boldsymbol{\rho} = \rho\hat{\boldsymbol{\rho}} = \mathbf{r}' - \mathbf{r}_s. \tag{7}$$

An analytical calculation of $\bar{\mathbf{W}}_E = \frac{1}{4\pi}\hat{\mathbf{R}} \times \int_0^1 \nabla \mathbf{E}^i_e(\tau \mathbf{R}) \exp(-jk\tau R)\, d\tau$ is carried out in [1, Appendix A] where $\mathbf{E}^i_e$ is the electric field due to an electric Hertzian dipole. Since the magnetic field from a magnetic Hertzian dipole is related to the electric field from an electric dipole by the principle of duality, it is seen that $\bar{\mathbf{W}}_H$ in (5) can be found from the result for $\bar{\mathbf{W}}_E$ in [1, (28)] as $\bar{\mathbf{W}}_H = Z^{-1}\bar{\mathbf{W}}_E$. In the expression for $\bar{\mathbf{W}}_E$, $\boldsymbol{\alpha}$ must be replaced by $Z^{-1}\boldsymbol{\alpha}_m$.

4. Numerical Example

Consider the rectangular planar PEC plate with dimensions 2λ by 3λ as shown in Fig. 1. The origin of the xyz coordinate system is located at one corner of the plate and the magnetic Hertzian dipole with dipole moment $\boldsymbol{\alpha}_m = 376(1, 1, 1)$ Vm is placed at (1λ, 2λ, 1λ). The observation points are located on the circular arc $r = 4\lambda$ in the $\phi = 50$ degrees plane with θ ranging from 0 to 90 degrees. For this configuration the Hertzian dipole is inside the cone with vertex at the observation point for $0 \le \theta \le 57.27$ degrees and outside the cone for $57.27 < \theta \le 90$ degrees. Fig. 2 shows the amplitudes of the x, y, and z components of the magnetic PO scattered field calculated from the traditional surface integral as well as from the line integral (6). As expected, perfect agreement is observed. Similar agreement is found for the phases.

References

[1] P. Meincke Johansen and O. Breinbjerg, "An Exact Line Integral Representation of the Physical Optics Scattered Field: The Case of a Perfectly Conducting Polyhedral Structure Illuminated by Electric Hertzian Dipoles," *IEEE Trans. Antennas Propagat.*, vol. 43, no. 7, pp. 689-696, July 1995.

[2] E. Martini, G. Pelosi, and G. Toso, "The physical optics field of a penetrable planar structure in terms of line integrals," Proceedings of journées internationales de Nice sur les antennes (JINA), pp. 218–221, Nov. 1998.

[3] D. S. Jones, *The Theory of Electromagnetism*, New York: Pergamon, 1964, p. 635.

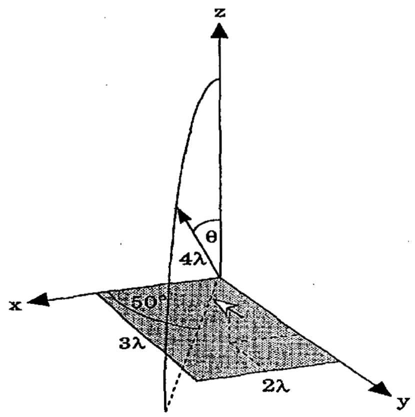

Figure 1: A 2λ by 3λ plate illuminated by a magnetic Hertzian dipole located at $(1\lambda, 2\lambda, 1\lambda)$ and with dipole moment $\boldsymbol{\alpha}_m = 376(1,1,1)$ Vm. The observation points are at $r = 4\lambda$ in the $\phi = 50$ degrees plane.

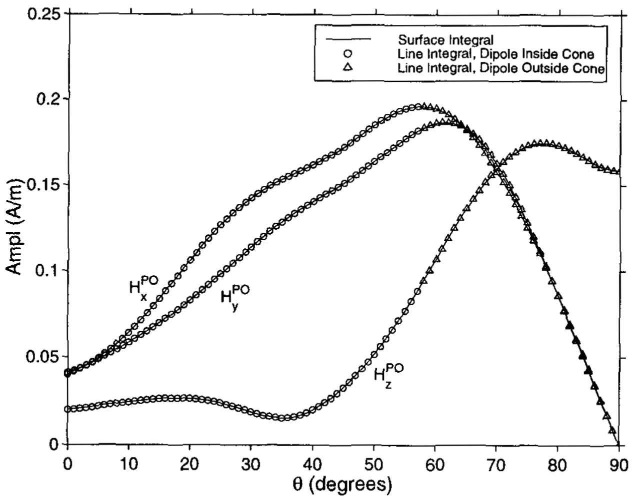

Figure 2: The amplitudes of the three rectangular components of the magnetic PO scattered field for the configuration shown in Fig. 1.